Chirality of exchange spin waves exposed: Scattering and emission from interfaces between antiferromagnetically coupled ferromagnets

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Abstract

We have studied the effect of chirality of the magnetization precession on the emission and scattering of spin waves from interfaces between ferromagnets with antiparallel magnetization. The need to match chirality of the magnetization precessions at such interfaces promotes coupling between propagating modes on one side of the interface and evanescent modes on the other. This leads to the following two major effects: (i) an asymmetrical (ultimately, unidirectional) emission of spin waves from such interfaces when they are driven by an elliptically polarized magnetic field, and (ii) a strong (ultimately, complete) suppression of the transmission of spin waves incident upon such interfaces. Our results are relevant to construction of spin-wave devices and more generally to interpretation of measurements and numerical simulations in the field of chiral magnonics.

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I. Introduction

As a pseudovector, the magnetization changes its sign upon time reversal. Hence, it is not surprising that chirality (or 'handedness') is inherent to dynamic magnetic phenomena. This includes a host of non-reciprocal phenomena observed in magneto-photonic [1], magneto-elastic [2] and magnonic structures [3,4]. In the latter, the chirality of spin waves [5] is usually related to either the magneto-dipole field [3,4,6] or the Dzyaloshinskii–Moriya interaction [7,8]. At the same time, the propagation of the most basic and 'usual' exchange spin waves, i.e. those with a parabolic dispersion relation dominated by the symmetric exchange interaction, is reciprocal. Upon a closer examination, one finds that the magnetic chirality is related to the existence of exponentially varying (decaying or growing) solutions of the exchange spin-wave dispersion relation – evanescent modes [9-13]. Notably, the handedness of precession of evanescent spin-wave modes is opposite to that of propagating modes [12], so that the propagating and evanescent modes at the same frequency complement each other. However, the exponential variation of evanescent modes in space makes them unphysical in unlimited magnonic media. In media with nonuniformities, such as interfaces [11-13] or magnetic domain walls [10], evanescent modes are physical but usually are either completely decoupled from or have only secondary effect on propagating modes.

Here, we demonstrate that the chirality of magnetic precession in ferromagnets emerges boldly in emission and scattering of exchange spin waves from interfaces between antiferromagnetically coupled, antiparallelly magnetized ferromagnets. When using the Schlömann-Wigen mechanism of spin-wave excitation,[14-15] the chirality of the emitted spin waves is determined by that of the incident microwave magnetic field. As a result, antiferromagnetically coupled interfaces pumped by an elliptically polarized microwave magnetic field emit spin waves asymmetrically, i.e. the intensities emitted in the opposite directions are different. In spin-wave scattering, the chirality of the secondary waves is determined by that of the incident spin wave. Thus, spin-wave scattering from an antiferromagnetically coupled interface can be efficiently controlled by a proper choice of the anisotropies of and therefore the precession ellipticity in the adjacent ferromagnetic media. We show two peculiar effects that occur when the precession ellipticity of the propagating mode in one of the two adjacent ferromagnets with coupled antiparallel magnetizations matches that of the evanescent mode in the other: a unidirectional spin-wave emission and a total spin-wave reflection. We give conditions for the effects to be observable across the whole spectrum of exchange spin waves rather than at a particular frequency value.

The paper is organized as follows. In the next Section (Section II), we present the main result of our study: the argument about the relation between the chirality of the incident stimulus (microwave magnetic field or spin wave) and the properties of the secondary spin waves emitted from the interface because of the excitation. In Section III, we present examples of application of this argument to some special cases of spin-wave emission and scattering from an interface between uniaxial ferromagnets. In Section IV, we present a semi-quantitative argument for how the phenomena should occur at an interface between biaxial ferromagnets. Finally, Section V is devoted to conclusions and outlook for applications of our findings.

II. Chirality of spin-wave modes at an interface between two uniaxial magnonic media

Let us begin by considering an interface between two semi-infinite uniaxial ferromagnetic media A and B (Fig. 1). The dynamics of their magnetizations $\boldsymbol{M}_{\mathrm{n}}$ is described using the linearized Landau-Lifshitz equation

$$\frac{\partial \boldsymbol{m}_{\mathrm{n}}}{\partial t} = \gamma \left[\boldsymbol{M}_{0,\mathrm{n}} \times \frac{\delta w_{\mathrm{n}}}{\delta \boldsymbol{m}_{\mathrm{n}}} \right] , \tag{1}$$

where t is time, γ is the gyromagnetic ratio, subscript $\mathrm{n} = \mathrm{A}, \mathrm{B}$ denotes quantities describing media A and B, $\boldsymbol{m}_{\mathrm{n}}$ are small perturbations of the static magnetization $\boldsymbol{M}_{0,\mathrm{n}}$, and the quadratic in $\boldsymbol{m}_{\mathrm{n}}$ magnetic energy density is

$$w_{\mathrm{n}} = \frac{1}{2} \lambda_{\mathrm{n}}^2 \left(\frac{\partial \boldsymbol{m}_{\mathrm{n}}}{\partial z} \right)^2 + \frac{1}{2} \beta_{\mathrm{n}} \left(m_{\mathrm{n},y}^2 + m_{\mathrm{n},z}^2 \right) . \tag{2}$$

Here, $\lambda_{\mathrm{n}} = \frac{\sqrt{2A_{\mathrm{n}}}}{M_{\mathrm{n}}}$ is the exchange length, A_{n} is the exchange constant, M_{n} is the saturation magnetization, and β_{n} is the anisotropy constant. The anisotropy easy axis is parallel to the interface. The Cartesian coordinate system is chosen so that its x axis is parallel to the easy axis, and its z axis is normal to the interface. The magnetization may be nonuniform only in the direction normal to the interface, i.e. we will only consider spin waves either normally incident onto the interface or emitted from it.

We assume for now that the magnetizations are rigidly coupled at the interface, leaving the case of a finite interlayer exchange coupling for the next section. The static magnetizations in the layers are uniform and given by vectors $\boldsymbol{M}_{\mathrm{n}} = (\sigma_{\mathrm{n}} M_{\mathrm{n}}, 0,0)$, where marker $\sigma_{\mathrm{n}} = \pm 1$ defines the equilibrium orientations of the magnetizations. The coupling may be either ferromagnetic ($\sigma_{\mathrm{A}} = \sigma_{\mathrm{B}}$) or antiferromagnetic ($\sigma_{\mathrm{A}} = -\sigma_{\mathrm{B}}$). The spin wave solutions are sought in form $\boldsymbol{m}_{\mathrm{n}} \propto e^{i(k_{\mathrm{n}} z - \omega t)}$, where the frequency ω is positively defined, and k_{n} are either wave numbers or exponential decay rates, depending on whether the solution is propagating or evanescent.

Projecting the Landau-Lifshitz equation on the yz plane, we obtain

$$\begin{aligned} \sigma_{\mathrm{n}} \omega_{\mathrm{n}} (\lambda_{\mathrm{n}}^2 k_{\mathrm{n}}^2 + \beta_{\mathrm{n}}) m_{\mathrm{n},y} + i\omega m_{\mathrm{n},z} = 0 \, , \\ \sigma_{\mathrm{n}} \omega_{\mathrm{n}} (\lambda_{\mathrm{n}}^2 k_{\mathrm{n}}^2 + \beta_{\mathrm{n}}) m_{\mathrm{n},z} - i\omega m_{\mathrm{n},y} = 0 \, , \end{aligned} \tag{3}$$

where we have introduced notations $\omega_{\mathrm{n}} = \gamma M_{\mathrm{n}} > 0$.

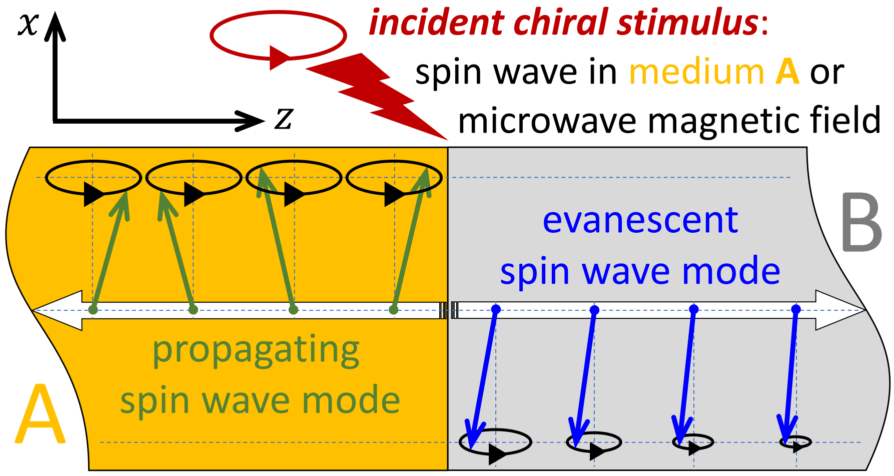

Fig. 1 The geometry of the problem is schematically shown for the case of an isolated interface between two semi-infinite magnonic media A and B magnetized parallel and antiparallel to the x axis, respectively. The media's magnetizations are coupled at the interface, and their precession chiralities are therefore the same. The precession chirality is determined by the incident chiral stimulus, which may be e.g. a chiral microwave magnetic field or a spin wave (in medium A in the shown case). Hence, the secondary spin wave modes emitted from the interface due to this stimulus have propagating character in one medium (medium A in the shown case) but are evanescent in the other (medium B in the shown case).

Introducing cyclic variables $m_{\mathrm{n},\pm} = m_{\mathrm{n},y} \pm i m_{\mathrm{n},z}$, we can re-write Eq. (3) as

$$\left(\lambda_{\mathrm{n}}^2 k_{\mathrm{n}}^2 + \beta_{\mathrm{n}} \pm \sigma_{\mathrm{n}} \frac{\omega}{\omega_{\mathrm{n}}}\right) m_{\mathrm{n},\pm} = 0\,, \tag{4}$$

from which immediately obtain the dispersion relations for solutions $m_{\mathrm{n},\pm}$ as

$$\lambda_{\mathrm{n}}^2 k_{\mathrm{n}}^2 + \beta_{\mathrm{n}} \pm \sigma_{\mathrm{n}} \frac{\omega}{\omega_{\mathrm{n}}} = 0\,. \tag{5}$$

Let us now recall that media A and B are both uniaxial and the precession of their magnetizations is therefore circularly polarized. Moreover, since the magnetizations at the interface are rigidly coupled, the chirality of their precession must be the same, irrespective of whether the static magnetizations are parallel or antiparallel. Hence, throughout the paper, we define chirality as the sense of rotation relative to the x axis of the laboratory coordinate system. That is, the rigid coupling means that if we have $m_{\mathrm{A},+} = 0$ and $m_{\mathrm{A},-} \neq 0$, then we must also have $m_{\mathrm{B},+} = 0$ and $m_{\mathrm{B},-} \neq 0$; and vice versa, if we have $m_{\mathrm{A},+} \neq 0$ and $m_{\mathrm{A},-} = 0$, then we must also have $m_{\mathrm{B},+} \neq 0$ and $m_{\mathrm{B},-} = 0$. As we will see shortly, the chirality of precession excited in each particular situation is then determined by the chirality of the external stimuli – either the incident spin wave (in scattering problems) or the microwave magnetic field (in emission problems).

Let us now formalize these conclusions for specific situations, starting from an interface with ferromagnetic coupling and $\sigma_\mathrm{A} = \sigma_\mathrm{B} = +1$. Then, for the case of $m_{\mathrm{n},+} = 0$ and $m_{\mathrm{n},-} \neq 0$, we have $m_{\mathrm{n},z} = i m_{\mathrm{n},y}$. Hence, assuming that $m_{\mathrm{n},y} \propto \mathrm{Re}\left(e^{-i\omega t}\right) = \cos\omega t$, we have $m_{\mathrm{n},z} \propto \mathrm{Re}\left(e^{i\left(\frac{\pi}{2}-\omega t\right)}\right) = \sin\omega t$. That is, our spin wave is right-circularly polarized: the vector of the dynamic magnetization rotates clockwise if one looks in the direction of the x axis, which coincides in this case with the direction of the static magnetization. For $m_{\mathrm{n},-} \neq 0$, Eqs. (4),(5) yield

$$k_\mathrm{n} = \pm k_{\mathrm{n,p}} \equiv \pm \frac{1}{\lambda_\mathrm{n}} \sqrt{\frac{\omega}{\omega_\mathrm{n}} - \beta_\mathrm{n}} \; . \tag{6}$$

From this, we see that the wave numbers k_n are real (here and in the following, we assume $\omega > \omega_\mathrm{n}\beta_\mathrm{n}$), and so, the $m_{\mathrm{n},-}$ solutions describe propagating modes (hence, the subscript ‘p’).

Following analogous considerations for the case of $m_{\mathrm{n},+} \neq 0$ and $m_{\mathrm{n},-} = 0$, we find

$$k_\mathrm{n} = \pm i k_{\mathrm{n,e}} \equiv \pm \frac{i}{\lambda_\mathrm{n}} \sqrt{\frac{\omega}{\omega_\mathrm{n}} + \beta_\mathrm{n}} \; , \tag{7}$$

so that the $m_{\mathrm{n},+}$ solutions describe evanescent modes (hence, the subscript ‘e’), which turn out to be left-circularly polarized: the vector of the dynamic magnetization rotates counterclockwise if one looks in the direction of the x axis.

When $\sigma_\mathrm{A} = \sigma_\mathrm{B} = -1$ (the magnetizations are parallel, but the media are both magnetized antiparallel to the x axis), the solutions $m_{\mathrm{n},+}$ and $m_{\mathrm{n},-}$ swap their roles: $m_{\mathrm{n},+}$ becomes propagating, while $m_{\mathrm{n},-}$ becomes evanescent. In $m_{\mathrm{n},+}$ ($m_{\mathrm{n},-}$), the vector of the dynamic magnetization rotates clockwise (counterclockwise) if one looks in the direction of the x axis, which is now antiparallel to the static magnetization. So, the sense of precession relative to the static magnetization is determined by whether the mode is propagating (clockwise precession) or evanescent (counterclockwise precession).

This consideration allows us to make important conclusions regarding the scattering and emission of spin waves from an interface between ferromagnetically coupled uniaxial media. In the scattering problem, the incident wave is a propagating mode: $m_{\mathrm{n},-}$ if $\sigma_\mathrm{A} = \sigma_\mathrm{B} = +1$, and $m_{\mathrm{n},+}$ if $\sigma_\mathrm{A} = \sigma_\mathrm{B} = -1$. The scattered waves are then also necessarily propagating modes of the same chirality as the incident wave. This scenario is consistent with those considered in earlier studies [16,17,]. In the emission problem, the emitted waves inherit chirality from the driving microwave magnetic field. If the driving field is polarized circularly with chirality coinciding with that of propagating modes, the latter will be emitted from the interface, as described e.g. in Refs. 18,19.

If the driving field's chirality matches that of the evanescent modes, the latter will be excited but will not propagate away from the interface due to their evanescent character. A linearly or elliptically polarized driving field can be represented as a superposition of left- and right-circularly polarized components and will therefore excite both propagating and evanescent modes. However, only propagating modes will "survive" at a distance from the interface.

Let us now turn to the case of antiferromagnetic coupling at the interface, starting from the case of $m_{\mathrm{n},+} = 0$ and $m_{\mathrm{n},-} \neq 0$. When $\sigma_{\mathrm{A}} = -\sigma_{\mathrm{B}} = +1$, we have $k_{\mathrm{A}} = \pm k_{\mathrm{A,p}}$ and $k_{\mathrm{B}} = \pm ik_{\mathrm{B,e}}$, i.e. the $m_{\mathrm{n},-}$ solution is a propagating mode in medium A and is evanescent in medium B. When $\sigma_{\mathrm{A}} = -\sigma_{\mathrm{B}} = -1$, we have $k_{\mathrm{A}} = \pm ik_{\mathrm{A,e}}$ and $k_{\mathrm{B}} = \pm k_{\mathrm{B,p}}$, i.e. the $m_{\mathrm{n},-}$ solution is an evanescent mode in medium A and is propagating in medium B. In the case of $m_{\mathrm{n},+} \neq 0$ and $m_{\mathrm{n},-} = 0$, the situation is reversed: when $\sigma_{\mathrm{A}} = -\sigma_{\mathrm{B}} = +1$, we have $k_{\mathrm{A}} = \pm ik_{\mathrm{A,e}}$ and $k_{\mathrm{B}} = \pm k_{\mathrm{B,p}}$, while we have $k_{\mathrm{A}} = \pm k_{\mathrm{A,p}}$ and $k_{\mathrm{B}} = \pm ik_{\mathrm{B,e}}$, when $\sigma_{\mathrm{A}} = -\sigma_{\mathrm{B}} = -1$.

As with the case of ferromagnetic coupling, let us make some preliminary observations regarding the scattering and emission of spin waves from an interface between antiferromagnetically coupled uniaxial ferromagnetic media. In the scattering problem, the chirality of the incident propagating mode will match chiralities of the reflected propagating mode (in the same medium) and of the transmitted evanescent mode (in the other medium, where the magnetization is antiparallel). Hence, the spin wave must be fully reflected from an isolated interface with antiferromagnetic coupling between two uniaxial ferromagnets. This scenario contradicts those considered e.g. in Refs. 20,21, where the chirality aspect seems to have been overlooked. In the emission problem, the chirality of a circularly polarized driving microwave magnetic field will be able to match the chirality of the propagating mode in only one of the two media, and the evanescent mode will be excited in the other medium. The emission will be unidirectional since the evanescent wave will decay quickly in space. As before, a linearly or elliptically polarized driving field will excite both propagating and evanescent modes, with only propagating modes "surviving" at a distance from the interface. However, unlike the case of the ferromagnetic interfacial coupling, the chiralities of the propagating modes emitted in opposite directions will be opposite and the mode's relative amplitude will be continuously tuneable by varying the polarization of the driving microwave magnetic field. Curiously, these observations do not require the two media to be different but only need them to be antiparallelly magnetized.

As an aside, we note that a driving field with frequency smaller than the uniform precession frequency will not excite any propagating modes. The driven precession near the interface will be described by the evanescent solutions on both side of the interface. If the magnetizations in the media are antiparallel, then the depth of penetration of the modes into the media will be different even if the two media are identical.

III. Case studies: Spin-wave emission and scattering coefficients

A. Spin-wave emission: An isolated interface between uniaxial magnonic media

Here, we formalize the considerations and results discussed in the previous section. To do this, we need to derive the high-frequency susceptibility of the two adjacent magnonic media. Adding the Gilbert damping term $\frac{\alpha_\mathrm{n}}{M_\mathrm{n}}\left[\boldsymbol{M}_{0,\mathrm{n}} \times \frac{\partial \boldsymbol{m}_\mathrm{n}}{\partial t}\right]$ and the uniform external microwave magnetic field $\boldsymbol{h}e^{-i\omega t}$ to the Landau-Lifshitz equation (1), we obtain in the frequency domain

$$
\begin{aligned}
(\omega_\mathrm{n}\beta_\mathrm{n} - i\omega\alpha_\mathrm{n})m_{\mathrm{n},y} + i\sigma_\mathrm{n}\omega m_{\mathrm{n},z} &= \omega_n h_y \ , \\
(\omega_\mathrm{n}\beta_\mathrm{n} - i\omega\alpha_\mathrm{n})m_{\mathrm{n},z} - i\sigma_\mathrm{n}\omega m_{\mathrm{n},y} &= \omega_n h_z \ .
\end{aligned}
\tag{8}
$$

Introducing variables $h_\pm = h_y \pm i h_z$, we obtain for the circular susceptibilities $\hat{\chi}_{\mathrm{n},\pm}$ (defined via $m_{\mathrm{n},\pm} = \hat{\chi}_{\mathrm{n},\pm} h_\pm$)

$$
\hat{\chi}_{\mathrm{n},\pm} = \frac{1}{\beta_\mathrm{n} \pm \sigma_\mathrm{n}\frac{\omega}{\omega_\mathrm{n}} - i\alpha_\mathrm{n}\frac{\omega}{\omega_\mathrm{n}}} \ . \tag{9}
$$

If the chirality of the pumping field matches that of precession in propagating spin waves, the combination $\pm\sigma_\mathrm{n}$ returns –1, and so

$$
\hat{\chi}_{\mathrm{n},\pm} = -\frac{1}{\frac{\omega}{\omega_\mathrm{n}} - \beta_\mathrm{n} + i\alpha_\mathrm{n}\frac{\omega}{\omega_\mathrm{n}}} \equiv -\frac{1}{\lambda_\mathrm{n}^2 k_{\mathrm{n,p}}^2} \ . \tag{10}
$$

If the chirality of the pumping field matches that of precession in evanescent spin waves, the combination $\pm\sigma_\mathrm{n}$ returns +1, and so

$$
\hat{\chi}_{\mathrm{n},\pm} = \frac{1}{\frac{\omega}{\omega_\mathrm{n}} + \beta_\mathrm{n} - i\alpha_\mathrm{n}\frac{\omega}{\omega_\mathrm{n}}} \equiv \frac{1}{\lambda_\mathrm{n}^2 k_{\mathrm{n,e}}^2} \ . \tag{11}
$$

Eqs. (10),(11) replace the definitions of $k_{\mathrm{n,p}}$ and $k_{\mathrm{n,e}}$, introduced earlier in Eqs. (6),(7), by accounting for the non-zero damping.

We describe spin waves using normalized magnetization, $\mu_{\mathrm{A},\pm} = C_{\mathrm{A},\pm}e^{-ik_\mathrm{A}z}$ and $\mu_{\mathrm{B},\pm} = C_{\mathrm{B},\pm}e^{+ik_\mathrm{B}z}$, where $C_{\mathrm{A},\pm}$ and $C_{\mathrm{B},\pm}$ are their complex amplitudes. Spin waves emitted from an interface between uniaxial media A and B pumped by field $h_\pm$ are given by solutions of the following algebraic system of equations (obtained as described in Ref. 18)

$$
\begin{gathered}
\sigma_\mathrm{A}\left(\mu_{\mathrm{B},\pm} - \frac{A_\mathrm{A}}{2A_\mathrm{AB}}\frac{\partial \mu_{\mathrm{A},\pm}}{\partial z}\right) - \sigma_\mathrm{B}\left(\mu_{\mathrm{A},\pm} + \frac{A_\mathrm{B}}{2A_\mathrm{AB}}\frac{\partial \mu_{\mathrm{B},\pm}}{\partial z}\right) = -\left(\frac{\sigma_\mathrm{A}\chi_{\mathrm{B},\pm}}{M_\mathrm{B}} - \frac{\sigma_\mathrm{B}\chi_{\mathrm{A},\pm}}{M_\mathrm{A}}\right)h_\pm \ , \\
\sigma_\mathrm{B}A_\mathrm{B}\frac{\partial \mu_{\mathrm{B},\pm}}{\partial z} - \sigma_\mathrm{A}A_\mathrm{A}\frac{\partial \mu_{\mathrm{A},\pm}}{\partial z} = 0 \ ,
\end{gathered}
\tag{12}
$$

where A_{AB} is the interlayer exchange constant, which has a negative value in the present case [22].

The case of ferromagnetically coupled media and a circularly polarized driving field with chirality matching that of propagating modes was considered in detail in Ref. 18. Hence, here we give results, obtained the same procedure, for the case of antiferromagnetically coupled magnonic media $\sigma_{\mathrm{A}} = -\sigma_{\mathrm{B}} = +1$. Let us assume that the incident driving field is right-circularly polarized, i.e. $h_{+} = 0$, $h_{-} \neq 0$. In this case, the amplitudes of the propagating and evanescent spin waves $\mu_{\mathrm{A},-} = C_{\mathrm{A},-} e^{-ik_{\mathrm{A,p}}z}$ and $\mu_{\mathrm{B},-} = C_{\mathrm{B},-} e^{-k_{\mathrm{B,e}}z}$ excited in medium A and medium B, respectively, are given by

$$C_{\mathrm{A},-} = -\frac{1}{1 - i\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{k_{\mathrm{B,e}}A_{\mathrm{B}}} + i\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{A_{\mathrm{AB}}}} \left(\frac{\chi_{\mathrm{A},-}}{M_{\mathrm{A}}} + \frac{\chi_{\mathrm{B},-}}{M_{\mathrm{B}}} \right) h_{-} \,,$$
$$C_{\mathrm{B},-} = -\frac{1}{1 + i\frac{k_{\mathrm{B,e}}A_{\mathrm{B}}}{k_{\mathrm{A,p}}A_{\mathrm{A}}} - \frac{k_{\mathrm{B,e}}A_{\mathrm{B}}}{A_{\mathrm{AB}}}} \left(\frac{\chi_{\mathrm{A},-}}{M_{\mathrm{A}}} + \frac{\chi_{\mathrm{B},-}}{M_{\mathrm{B}}} \right) h_{-} \,, \tag{13}$$

where

$$\frac{\chi_{\mathrm{A},-}}{M_{\mathrm{A}}} + \frac{\chi_{\mathrm{B},-}}{M_{\mathrm{B}}} = \frac{1}{M_B\left(\frac{\omega}{\omega_{\mathrm{B}}} + \beta_{\mathrm{B}} - i\alpha_{\mathrm{B}}\frac{\omega}{\omega_{\mathrm{B}}}\right)} - \frac{1}{M_{\mathrm{A}}\left(\frac{\omega}{\omega_{\mathrm{A}}} - \beta_{\mathrm{A}} + i\alpha_{\mathrm{A}}\frac{\omega}{\omega_{\mathrm{A}}}\right)} = \frac{1}{M_{\mathrm{B}}\lambda_{\mathrm{B}}^2 k_{\mathrm{B,e}}^2} - \frac{1}{M_{\mathrm{A}}\lambda_{\mathrm{A}}^2 k_{\mathrm{A,p}}^2} \,. \tag{14}$$

If we switch the chirality of the driving field, i.e. for $h_{+} \neq 0$, $h_{-} = 0$, the media will switch their roles: propagating spin waves (i.e. those of practical interest) will be generated into medium B, i.e. $\mu_{\mathrm{B},+} = C_{\mathrm{B},+} e^{ik_{\mathrm{B,p}}z}$, while spin waves emitted into medium A will be evanescent, i.e. $\mu_{\mathrm{A},+} = C_{\mathrm{A},+} e^{k_{\mathrm{A,e}}z}$, so that

$$C_{\mathrm{A},+} = -\frac{1}{1 + i\frac{k_{\mathrm{A,e}}A_{\mathrm{A}}}{k_{\mathrm{B,p}}A_{\mathrm{B}}} - \frac{k_{\mathrm{A,e}}A_{\mathrm{A}}}{A_{\mathrm{AB}}}} \left(\frac{\chi_{\mathrm{A},-}}{M_{\mathrm{A}}} + \frac{\chi_{\mathrm{B},-}}{M_{\mathrm{B}}} \right) h_{+} \,,$$
$$C_{\mathrm{B},+} = -\frac{1}{1 - i\frac{k_{\mathrm{B,p}}A_{\mathrm{B}}}{k_{\mathrm{A,e}}A_{\mathrm{A}}} + i\frac{k_{\mathrm{B,p}}A_{\mathrm{B}}}{A_{\mathrm{AB}}}} \left(\frac{\chi_{\mathrm{A},-}}{M_{\mathrm{A}}} + \frac{\chi_{\mathrm{B},-}}{M_{\mathrm{B}}} \right) h_{+} \,, \tag{15}$$

where

$$\frac{\chi_{\mathrm{A},+}}{M_{\mathrm{A}}} + \frac{\chi_{\mathrm{B},+}}{M_{\mathrm{B}}} = \frac{1}{M_{\mathrm{A}}\left(\frac{\omega}{\omega_{\mathrm{A}}} + \beta_{\mathrm{A}} - i\alpha_{\mathrm{A}}\frac{\omega}{\omega_{\mathrm{A}}}\right)} - \frac{1}{M_{\mathrm{B}}\left(\frac{\omega}{\omega_{\mathrm{B}}} - \beta_{\mathrm{B}} + i\alpha_{\mathrm{B}}\frac{\omega}{\omega_{\mathrm{B}}}\right)} = \frac{1}{M_{\mathrm{A}}\lambda_{\mathrm{A}}^2 k_{\mathrm{A,e}}^2} - \frac{1}{M_{\mathrm{B}}\lambda_{\mathrm{B}}^2 k_{\mathrm{B,p}}^2} \,. \tag{16}$$

Similarly, a simultaneous switching of the media's magnetizations, so that $\sigma_{\mathrm{A}} = -\sigma_{\mathrm{B}} = -1$, does not explicitly affect Eq. (12) but modifies (for unchanged chirality of driving field) the character of modes excited in media A and B. The consequence of the latter modification is described by swapping $k_{\mathrm{n,p}}^2 \leftrightarrow -k_{\mathrm{n,e}}^2$ in Eqs. (13)-(16).

The strength of the interlayer exchange coupling does not modify the propagating or evanescent character of spin waves emitted from the interface. Hence, the chiral effects remain.

However, Eqs. (15)-(16) show that a weakened interlayer exchange interaction leads to a weaker emission of both the propagating and evanescent modes.

B. Spin-wave emission: A layer sandwiched between two uniaxial magnonic media

The case of emission from a ferromagnetic layer embedded within and ferromagnetically coupled to another magnonic medium was considered in detail in Ref. 19. Hence, as before, we limit ourselves to the results, obtained using the same procedure, for the case of an antiferromagnetic interfacial coupling and $\sigma_{\mathrm{A}} = -\sigma_{\mathrm{B}} = +1$. The sandwiched layer is made of material B and has thickness d. For pumping with field of right-circular polarization, $h_{+} = 0$, $h_{-} \neq 0$, the spin-wave solutions have form

$$\begin{aligned} \mu_{\mathrm{A1},-} &= C_{\mathrm{A1},-} e^{-ik_{\mathrm{A,p}}\left(z+\frac{d}{2}\right)} , \\ \mu_{\mathrm{B},-} &= B_1 \mathrm{ch}\left(k_{\mathrm{B,e}} z\right) + B_2 \mathrm{sh}\left(k_{\mathrm{B,e}} z\right) , \\ \mu_{\mathrm{A2},-} &= C_{\mathrm{A2},-} e^{ik_{\mathrm{A,p}}\left(z-\frac{d}{2}\right)} , \end{aligned} \tag{17}$$

where subscripts A1 and A2 refer to the semi-infinite media on the left ($z < -\frac{d}{2}$) and on the right ($z > +\frac{d}{2}$) from the layer B ($-\frac{d}{2} < z < +\frac{d}{2}$), respectively.

Substituting Eq. (17) into systems (12) for each of the interfaces, we obtain

$$\begin{aligned} & C_{\mathrm{A1},-}\left(1 + i\frac{A_{\mathrm{A}} k_{\mathrm{A,p}}}{2A_{\mathrm{AB}}}\right) + B_1\left(\mathrm{ch}\left(\frac{k_{\mathrm{B,e}} d}{2}\right) - \frac{A_{\mathrm{B}} k_{\mathrm{B,e}}}{2A_{\mathrm{AB}}}\mathrm{sh}\left(\frac{k_{\mathrm{B,e}} d}{2}\right)\right) \\ & -B_2\left(\mathrm{sh}\left(\frac{k_{\mathrm{B,e}} d}{2}\right) - \frac{A_{\mathrm{B}} k_{\mathrm{B,e}}}{2A_{\mathrm{AB}}}\mathrm{ch}\left(\frac{k_{\mathrm{B,e}} d}{2}\right)\right) = -\left(\frac{\chi_{\mathrm{A},-}}{M_{\mathrm{A}}} + \frac{\chi_{\mathrm{B},-}}{M_{\mathrm{B}}}\right) h_{-} , \\ & B_1\left(\mathrm{ch}\left(\frac{k_{\mathrm{B,e}} d}{2}\right) - \frac{A_{\mathrm{B}} k_{\mathrm{B,e}}}{2A_{\mathrm{AB}}}\mathrm{sh}\left(\frac{k_{\mathrm{B,e}} d}{2}\right)\right) + B_2\left(\mathrm{sh}\left(\frac{k_{\mathrm{B,e}} d}{2}\right) - \frac{A_{\mathrm{B}} k_{\mathrm{B,e}}}{2A_{\mathrm{AB}}}\mathrm{ch}\left(\frac{k_{\mathrm{B,e}} d}{2}\right)\right) \\ & +C_{\mathrm{A2},-}\left(1 + i\frac{A_{\mathrm{A}} k_{\mathrm{A,p}}}{2A_{\mathrm{AB}}}\right) = -\left(\frac{\chi_{\mathrm{A},-}}{M_{\mathrm{A}}} + \frac{\chi_{\mathrm{B},-}}{M_{\mathrm{B}}}\right) h_{-} , \\ & -ik_{\mathrm{A,p}} A_{\mathrm{A}} C_{\mathrm{A1},-} + k_{\mathrm{B,e}} A_{\mathrm{B}}\left(-B_1 \mathrm{sh}\left(\frac{k_{\mathrm{B,e}} d}{2}\right) + B_2 \mathrm{ch}\left(\frac{k_{\mathrm{B,e}} d}{2}\right)\right) = 0 , \\ & k_{\mathrm{B,e}} A_{\mathrm{B}}\left(B_1 \mathrm{sh}\left(\frac{k_{\mathrm{B,e}} d}{2}\right) + B_2 \mathrm{ch}\left(\frac{k_{\mathrm{B,e}} d}{2}\right)\right) + ik_{\mathrm{A,p}} A_{\mathrm{A}} C_{\mathrm{A2},-} = 0 . \end{aligned} \tag{18}$$

Solving this system of equations, we obtain for the complex amplitudes of propagating spin waves emitted into medium A:

$$C_{\mathrm{A1},-} = C_{\mathrm{A2},-} = -\frac{1}{1 - i\frac{k_{\mathrm{A,p}} A_{\mathrm{A}}}{k_{\mathrm{B,e}} A_{\mathrm{B}}}\mathrm{cth}\left(\frac{k_{\mathrm{B,e}} d}{2}\right) + i\frac{A_{\mathrm{A}} k_{\mathrm{A,p}}}{A_{\mathrm{AB}}}}\left(\frac{\chi_{\mathrm{A},-}}{M_{\mathrm{A}}} + \frac{\chi_{\mathrm{B},-}}{M_{\mathrm{B}}}\right) h_{-} . \tag{19}$$

As expected, in the limit of large layer thicknesses, we have $\operatorname{cth}\left(\frac{k_{\mathrm{B,e}}d}{2}\right) \to 1$, and so, Eq. (18) reduces to the first of Eq. (13), i.e. to the result obtained for an isolated interface. At high driving frequencies, we have both $\operatorname{cth}\left(\frac{k_{\mathrm{B,e}}d}{2}\right) \to 1$ and $\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{k_{\mathrm{B,e}}A_{\mathrm{B}}} \to \frac{A_{\mathrm{A}}}{A_{\mathrm{B}}}$.

A microwave magnetic field of left-circular polarization, $h_+ \neq 0$, $h_- = 0$, couples to propagating spin-wave solutions inside and evanescent modes outside of the layer. A simple calculation (not shown) shows that, as a result of such pumping, standing spin waves with a symmetric field distribution are formed in the layer $(-\frac{d}{2} < z < \frac{d}{2})$:

$$\mu_{\mathrm{B}} = B\cos(k_{\mathrm{B,p}}z)\left(\frac{\chi_{\mathrm{A},+}}{M_{\mathrm{A}}} + \frac{\chi_{\mathrm{B},+}}{M_{\mathrm{B}}}\right)h_- \ , \tag{20}$$

where $B = \frac{1}{\cos\left(\frac{k_{\mathrm{B,p}}d}{2}\right) - \frac{k_{\mathrm{B,p}}A_{\mathrm{B}}}{k_{\mathrm{A,e}}A_{\mathrm{A}}}\sin\left(\frac{k_{\mathrm{B,p}}d}{2}\right) + \frac{k_{\mathrm{B,p}}A_{\mathrm{B}}}{A_{\mathrm{AB}}}\sin\left(\frac{k_{\mathrm{B,p}}d}{2}\right)}$ and $\frac{\chi_{\mathrm{A},+}}{M_{\mathrm{A}}} + \frac{\chi_{\mathrm{B},+}}{M_{\mathrm{B}}}$ is defined by Eq. (16).

C. Spin-wave scattering: An isolated interface between uniaxial magnonic media

Let us consider an interface at $z = 0$ between semi-infinite uniaxial media A and B. The media are coupled antiferromagnetically, so that $\sigma_{\mathrm{A}} = -\sigma_{\mathrm{B}} = +1$. We consider a spin wave, $1 \cdot e^{ik_{\mathrm{A,p}}z}$, propagating in medium A: the wave has unit amplitude, is right-circularly polarized, i.e. $m_{\mathrm{A},+} = 0$ and $m_{\mathrm{A},-} \neq 0$, and is incident upon the interface from the left. As discussed before, the reflected and transmitted modes must have the same, i.e. right-circular, polarization. The reflected spin wave is $r \cdot e^{-ik_{\mathrm{A,p}}z}$ (medium A, propagating mode), where r is the reflection coefficient. The transmitted wave is $\tau \cdot e^{-k_{\mathrm{B,e}}z}$ (medum B, evanescent mode), where τ is the transmission coefficient. At the interface, the incident, reflected and transmitted waves must satisfy boundary conditions given by Eq. (12) with $h_\pm = 0$. This yields

$$r = -\frac{1 + i\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{k_{\mathrm{B,e}}A_{\mathrm{B}}} - i\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{A_{\mathrm{AB}}}}{1 - i\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{k_{\mathrm{B,e}}A_{\mathrm{B}}} + i\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{A_{\mathrm{AB}}}}, \quad \tau = \frac{2i\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{k_{\mathrm{B,e}}A_{\mathrm{B}}}}{1 - i\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{k_{\mathrm{B,e}}A_{\mathrm{B}}} + i\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{A_{\mathrm{AB}}}} . \tag{21}$$

Let us note that $|r| = 1$, and the spin wave is therefore fully reflected (irrespective of the value of A_{AB}), with a phase shift that depends on the wave's frequency. Let us also recall that τ is the amplitude of an evanescent mode, which does not carry energy away from the interface.

D. Spin-wave scattering: A layer sandwiched between two uniaxial magnonic media

Let us consider a layer $(-\frac{d}{2} < z < \frac{d}{2})$ of uniaxial material B sandwiched between and antiferromagnetically coupled to two semi-infinite uniaxial media A, so that $\sigma_{\mathrm{A}} = -\sigma_{\mathrm{B}} = +1$. Because of the need to match the precession chirality, the layer plays a role of a barrier, through which the energy tunnels in form of evanescent modes. So, we seek spin-wave solutions as

$$\mu_{\mathrm{A1}} = 1e^{ik_{\mathrm{A,p}}\left(z+\frac{d}{2}\right)} + re^{-ik_{\mathrm{A,p}}\left(z+\frac{d}{2}\right)} \ ,$$
$$\mu_{\mathrm{B}} = B_1 \mathrm{ch}\left(k_{\mathrm{B,e}} z\right) + B_2 \mathrm{sh}\left(k_{\mathrm{B,e}} z\right) \ , \tag{22}$$
$$\mu_{\mathrm{A2}} = \tau e^{ik_{\mathrm{A,p}}\left(z-\frac{d}{2}\right)} \ .$$

Applying the boundary conditions at the layer interfaces, we obtain the following system of equations:

$$\begin{gathered}
\left(1 - i\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{2A_{\mathrm{AB}}}\right) + \left(1 + i\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{2A_{\mathrm{AB}}}\right) r + B_1\left(\mathrm{ch}\left(\frac{k_{\mathrm{B,e}}d}{2}\right) - \frac{k_{\mathrm{B,e}}A_{\mathrm{B}}}{2A_{\mathrm{AB}}}\mathrm{sh}\left(\frac{k_{\mathrm{B,e}}d}{2}\right)\right) - \\
B_2\left(-\mathrm{sh}\left(\frac{k_{\mathrm{B,e}}d}{2}\right) + \frac{k_{\mathrm{B,e}}A_{\mathrm{B}}}{2A_{\mathrm{AB}}}\mathrm{ch}\left(\frac{k_{\mathrm{B,e}}d}{2}\right)\right) = 0 \ , \\
B_1\left(\mathrm{ch}\left(\frac{k_{\mathrm{B,e}}d}{2}\right) - \frac{k_{\mathrm{B,e}}A_{\mathrm{B}}}{2A_{\mathrm{AB}}}\mathrm{sh}\left(\frac{k_{\mathrm{B,e}}d}{2}\right)\right) + B_2\left(\mathrm{sh}\left(\frac{k_{\mathrm{B,e}}d}{2}\right) - \frac{k_{\mathrm{B,e}}A_{\mathrm{B}}}{2A_{\mathrm{AB}}}\mathrm{ch}\left(\frac{k_{\mathrm{B,e}}d}{2}\right)\right) + \\
\tau\left(1 + i\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{2A_{\mathrm{AB}}}\right) = 0 \ , \\
-ik_{\mathrm{A,p}}A_{\mathrm{A}}r - k_{\mathrm{B,e}}A_{\mathrm{B}}B_1\mathrm{sh}\left(\frac{k_{\mathrm{B,e}}d}{2}\right) + k_{\mathrm{B,e}}A_{\mathrm{B}}B_2\mathrm{ch}\left(\frac{k_{\mathrm{B,e}}d}{2}\right) = -ik_{\mathrm{A,p}}A_{\mathrm{A}} \ , \\
k_{\mathrm{B,e}}A_{\mathrm{B}}B_1\mathrm{sh}\left(\frac{k_{\mathrm{B,e}}d}{2}\right) + k_{\mathrm{B,e}}A_{\mathrm{B}}B_2\mathrm{ch}\left(\frac{k_{\mathrm{B,e}}d}{2}\right) + ik_{\mathrm{A,p}}A_{\mathrm{A}}\tau = 0 \ .
\end{gathered} \tag{23}$$

From this system, we obtain for the reflection and transmission coefficients for a wave incident from the left

$$\begin{gathered}
r = \frac{(\xi^2+1)\mathrm{sh}(k_{\mathrm{B,e}}d) - 2\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{A_{\mathrm{AB}}}\xi\mathrm{ch}(k_{\mathrm{B,e}}d) + \left(\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{A_{\mathrm{AB}}}\right)^2\mathrm{sh}(k_{\mathrm{B,e}}d)}{2i\xi\mathrm{ch}(k_{\mathrm{B,e}}d) + (\xi^2-1)\mathrm{sh}(k_{\mathrm{B,e}}d) - 2\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{A_{\mathrm{AB}}}\left(\xi\mathrm{ch}(k_{\mathrm{B,e}}d) + i\mathrm{sh}(k_{\mathrm{B,e}}d)\right) + \left(\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{A_{\mathrm{AB}}}\right)^2\mathrm{sh}(k_{\mathrm{B,e}}d)} \ , \\
\tau = \frac{2i\xi}{2i\xi\mathrm{ch}(k_{\mathrm{B,e}}d) + (\xi^2-1)\mathrm{sh}(k_{\mathrm{B,e}}d) - 2\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{A_{\mathrm{AB}}}\left(i\mathrm{sh}(k_{\mathrm{B,e}}d) + \xi\mathrm{ch}(k_{\mathrm{B,e}}d)\right) + \left(\frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{A_{\mathrm{AB}}}\right)^2\mathrm{sh}(k_{\mathrm{B,e}}d)} \ ,
\end{gathered} \tag{24}$$

where we have denoted $\xi = \frac{k_{\mathrm{A,p}}A_{\mathrm{A}}}{k_{\mathrm{B,e}}A_{\mathrm{B}}}$. The scattering coefficients for spin waves incident from opposite directions have equal absolute values, i.e. the system cannot be used as an isolator,[23,24] in contrast to magnonic devices discussed e.g. in Refs. 4,25.

IV. Discussion

A legitimate question arising from the results presented so far concerns spin-wave emission and scattering from interfaces between antiferromagnetically coupled media that are biaxial, so that the precession in each medium is elliptically polarized. Then, both propagating and evanescent modes need to be accounted for at each interface, e.g. as in Ref. 12. The corresponding algebra becomes quite complex, and the results are not worthwhile presenting unless a specific experimental situation is considered. Hence, we limit ourselves to a qualitative discussion of spin-wave scattering and emission from an isolated interface between two antiferromagnetically coupled biaxial ferromagnetic media, which however allows us to expose the underpinning physics.

Let us assume that the interfaced ferromagnets are biaxial, so that their magnetic energy density is

$$w_{\mathrm{n}} = \frac{1}{2}\lambda_{\mathrm{n}}^2 \left(\frac{\partial \boldsymbol{m}_{\mathrm{n}}}{\partial z}\right)^2 + \frac{1}{2}\beta_{y,\mathrm{n}} m_{\mathrm{n},y}^2 + \frac{1}{2}\beta_{z,\mathrm{n}} m_{\mathrm{n},z}^2 , \tag{25}$$

where $\beta_{y,\mathrm{n}} = \beta_{\mathrm{n}} - \delta\beta_{\mathrm{n}}$ and $\beta_{z,\mathrm{n}} = \beta_{\mathrm{n}} + \delta\beta_{\mathrm{n}}$ are the constants of the biaxial anisotropy in ferromagnets $\mathrm{n} = \mathrm{A}, \mathrm{B}$. Parameter $\beta_{\mathrm{n}} = \frac{\beta_{y,\mathrm{n}} + \beta_{z,\mathrm{n}}}{2}$ is their average anisotropy strength, while parameter $\delta\beta_{\mathrm{n}} = \frac{\beta_{z,\mathrm{n}} - \beta_{y,\mathrm{n}}}{2}$ describes the degree to which the ferromagnets deviate from having uniaxial anisotropy. In the rest of this paper, we assume the limit of zero damping, for simplicity.

Using Eq. (25), we obtain for the dispersion relation

$$\lambda_{\mathrm{n}}^2 k_{\mathrm{n},{\mathrm{p} \atop \mathrm{e}}}^2 = -\beta_{\mathrm{n}} \pm \sqrt{\left(\frac{\omega}{\omega_{\mathrm{n}}}\right)^2 + \delta\beta_{\mathrm{n}}^2} , \tag{26}$$

where the "+" and "–" signs in front of the radical correspond to the propagating and evanescent modes, respectively. The ellipticities of the propagating and evanescent modes are then given by

$$\eta_{\mathrm{n},{\mathrm{p} \atop \mathrm{e}}} = \sigma_{\mathrm{n}} \frac{-\delta\beta_{\mathrm{n}} \pm \sqrt{\left(\frac{\omega}{\omega_{\mathrm{n}}}\right)^2 + \delta\beta_{\mathrm{n}}^2}}{\frac{\omega}{\omega_{\mathrm{n}}}} = \sigma_{\mathrm{n}} \frac{\frac{\omega}{\omega_{\mathrm{n}}}}{+\delta\beta_{\mathrm{n}} \pm \sqrt{\left(\frac{\omega}{\omega_{\mathrm{n}}}\right)^2 + \delta\beta_{\mathrm{n}}^2}} . \tag{27}$$

From Eq. (27), we see that $\eta_{\mathrm{n,e}}\eta_{\mathrm{n,p}} = -1$ [11,12]. As for uniaxial magnets, this shows that the precession chiralities are opposite for propagating and evanescent modes. In addition, this means that the precession ellipses of the propagating and evanescent modes are rotated by 90 degrees relative to each other. This suggests a way to generalize the effects of full reflection and unidirectional emission from interfaces between antiferromagnetically coupled ferromagnets discussed in Section II for uniaxial anisotropy to the case of biaxial ferromagnets: the effects occur for all frequencies across the spectrum if $\delta\beta_{\mathrm{A}} = -\delta\beta_{\mathrm{B}}$ and $\omega_{\mathrm{A}} = \omega_{\mathrm{B}}$. Otherwise, there is just one such frequency, and both propagating and evanescent modes are emitted from the interface when it is excited either by a suitably polarized microwave magnetic field or by an incident spin wave.

It is also relatively straightforward to generalize our calculations and discussion to the case of oblique spin wave incidence or to that of spin wave emission due to a microwave field that is nonuniform in the interface plane. Qualitatively, the need to match chirality of precession at the interface remains the dominant factor. On the side of the interface where the chirality requires the wave to be evanescent, the overall wave vector must be imaginary. This means that an increase of the real component of the wave vector parallel to the interface leads to a corresponding increase of its imaginary component orthogonal to the interface, i.e. to a stronger localization of the mode near the interface.

V. Conclusions

The main premise of this paper, belonging to the topic of chiral magnonics, is the need to match chirality of the magnetization precessions at interfaces. In the emphasized here specific case of antiparallel alignment of the static magnetization, this chirality matching promotes coupling between propagating modes on one side of the interface and evanescent modes on the other. This leads to the following two major effects: (i) an asymmetrical (ultimately, unidirectional) emission of spin waves from such interfaces driven by an elliptically polarized magnetic field, and (ii) a strong (ultimately, complete) suppression of the spin-wave transmission through such interfaces. We note that the problems considered in our paper are complementary to that of calculation of the spectrum of surface and interface spin waves in magnetic multilayers, which was studied in depth e.g. in Refs. 26-29.

Notably, albeit exchange spin waves are used in our argument, the physics underpinning our predictions remains valid in the more general case of dipole-exchange spin waves [30,31]. However, the corresponding analytical theory is drastically more complex and less transparent, so that one might need instead to employ numerical calculations [30] to describe realistic experimental arrangements. As to the exchange spin-wave models presented in Section III, they are given here to remedy the results from Refs. 20,21, where the chirality aspect appears to have been overlooked. Our results are also relevant to the model from Ref. 10, the authors of which considered the spin-wave emission from a domain wall driven by a linearly (rather than elliptically) polarized microwave magnetic field. However, the need to consider a finite domain wall width places the system from Ref. 10 outside the scope of our relatively simple formalism. Nonetheless, our results provide interpretation for the "magnon blocking effect" observed numerically in Ref. 32 and relevant to the "magnon valve effect" from Ref. 33. However, a systematic application of our findings to systems from the references above is left for future studies. At last, but not least, we note that by measuring the relative spin-wave power emitted in opposite directions from interfaces between ferromagnets with antiparallel magnetization provides a method of measuring the ellipticity of the incident driving microwave magnetic field.

Data Availability

Data sharing is not applicable to this article as no new data were created or analyzed in this purely theoretical study.

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